Pion absorption and rescattering in the ANP model revisited

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Single pion leptoproduction in the region of the $(3,3)$ resonance is currently of high interest for at least two reasons: (i) These reactions constitute an important part of the total cross section in low energy reactions and are utilized to detect neutrino oscillations in current and future long baseline experiments. (ii) Intranuclear rescattering of the pions in heavy nuclei results in interesting and sizable modifications of the free nucleon cross sections which are testable in electroproduction experiments. In this article we give a basic introduction to the pion multiple scattering model of Adler, Nussinov, and Paschos (ANP) with special emphasis on pion absorption. We also estimate the probability of multiple scattering.

1. Introduction

Neutrino nucleus scattering will be an important tool in future long baseline (LBL) experiments to precisely measure neutrino parameters like neutrino masses and mixing angles. These experiments will use neutrino beams with energies of the order $\mathcal{O}(1\text{ GeV})$ where quasi-elastic reactions (QE), single pion resonance production (RES) and deep inelastic scattering (DIS) are all important. Moreover, due to the small neutrino cross sections heavy targets like oxygen, argon or iron have to be used.

In the following we will deal with single pion resonance production, i.e., with the reactions

$$\nu + T \to l + T' + \pi^{\pm,0} \tag{1}$$

where T is a nuclear target (${}_8O^{16}$, ${}_{18}Ar^{40}$, ${}_{26}Fe^{56}$) and T' a final nuclear state.

At this stage we make the basic assumption that the reaction (1) can be described by two *independent* steps [1]:

1. <u>Single pion production in νN scattering:</u>
   In this first step a pion is produced by the scattering of the incoming neutrino off a nucleon in the target, such that nuclear corrections due to the Pauli principle and the Fermi motion of the nucleons should be taken into account.
2. <u>Multiple scattering of pions:</u>
   Once the pion has been produced it will travel through the nucleus. During this journey the pion can have several rescatterings in which it can change its charge or be absorbed.

Step 2 can be described by a 3×3 charge exchange matrix M which depends only on the properties of the target [modeled by a charge density profile $\rho(r)$] but which is independent of the identities of the leptons in step 1. It should be noted that the above assumption of two independent steps generates predictive power, since the formalism can be applied to charged current (CC) and neutral current (NC) neutrino- and electroproduction of pions in the resonance region.

Absorbing the Pauli suppression factor of step 1 into the normalization of M, the measurable final distributions (cross sections) of pions $(\pi^+,\pi^0,\pi^-)_f$ can be related to the initial distributions $(\pi^+,\pi^0,\pi^-)_i$ for a *free* target in the simple form

$$\underbrace{\begin{pmatrix} \dfrac{\mathrm{d}\sigma({}_ZT^A;\pi^+)}{\mathrm{d}Q^2\mathrm{d}W} \\ \dfrac{\mathrm{d}\sigma({}_ZT^A;\pi^0)}{\mathrm{d}Q^2\mathrm{d}W} \\ \dfrac{\mathrm{d}\sigma({}_ZT^A;\pi^-)}{\mathrm{d}Q^2\mathrm{d}W} \end{pmatrix}}_{\text{nuclear target}} = M \underbrace{\begin{pmatrix} \dfrac{\mathrm{d}\sigma(N_T;\pi^+)}{\mathrm{d}Q^2\mathrm{d}W} \\ \dfrac{\mathrm{d}\sigma(N_T;\pi^0)}{\mathrm{d}Q^2\mathrm{d}W} \\ \dfrac{\mathrm{d}\sigma(N_T;\pi^-)}{\mathrm{d}Q^2\mathrm{d}W} \end{pmatrix}}_{\text{free nucleon}} \tag{2}$$

with

$$\frac{\mathrm{d}\sigma(N_T;\pm0)}{\mathrm{d}Q^2\mathrm{d}W} = Z\frac{\mathrm{d}\sigma(p;\pm0)}{\mathrm{d}Q^2\mathrm{d}W} + (A-Z)\frac{\mathrm{d}\sigma(n;\pm0)}{\mathrm{d}Q^2\mathrm{d}W}$$

where the free nucleon cross sections are averaged over the Fermi momentum of the nucleons. The matrix M depends on the target material and the final state kinematic variables, i.e. $M = M[T;Q^2,W]$.

In the following we will restrict the discussion to isoscalar targets and refer the reader to ref. [2] for targets with a neutron excess. The charge exchange matrix M for isoscalar targets can be parameterized by three independent parameters A_p, d, and c:

$$M = A_p \begin{pmatrix} 1-c-d & d & c \\ d & 1-2d & d \\ c & d & 1-c-d \end{pmatrix}. \quad (3)$$

So far there is no direct experimental information on these three parameters available. On the other hand a model for pion multiple scattering by Adler, Nussinov and Paschos (ANP model) [1] allows to calculate these parameters and (averaged) charge exchange matrices for oxygen, argon and iron targets can be found in [3].

These matrices are useful to calculate CC and NC single pion production in the (3,3) resonance region induced by neutrinos [3,4,5,6] and electrons [7] and the rescattering effects are predicted to be quite large. For example, the pion energy spectra for π^0's produced on oxygen targets in NC neutrino production will be reduced by up to 40% whereas the analogous spectra for the charged pions remain almost unchanged compared to the free nucleon cross sections [3]. Similar results are predicted for the W-distributions for 1-pion resonance production in electron-oxygen scattering [7]. For the π^0 the W-distribution will be reduced by about 50% while the production of π^+ remains almost unchanged compared to the free case since the decrease due to absorption is widely compensated by an increase due to charge exchange. This marked signal of rescattering effects should be easily testable in low energy electron scattering experiments, for example at JLAB.

The rest of this paper is organized as follows. In the next section we give an introduction to the ANP model. In Sec. 3 we discuss the normalization of pion absorption which we fix by experimental information. In Sec. 4 we discuss the dynamics of the ANP model in more detail. Finally in Sec. 5 we summarize the main results.

2. The ANP model

In this section we introduce the multiple scattering model by Adler, Nussinov and Paschos [1]. Due to space limitations the presentation will be short and we refer the reader to Ref. [1] for a thorough discussion of the omitted details.

The main ingredients of the model are as follows: (i) The target nucleus is taken to be a collection of independent nucleons distributed spatially according to a charge density profile $\rho(r)$. (ii) The nucleons are regarded to be fixed within the nucleus neglecting Fermi motion and nucleon recoil effects. This implies that the pion energy E_π does not change in the elastic scatterings. Furthermore the target is assumed to stay isotopically neutral ($Z = N$) during the multiple scattering processes. (iii) The pion interactions in the nucleus are assumed to be incoherent πN reactions taking place in the Δ resonance region. Since pion production and more complex channels are closed in this region the rescattering processes can be described by two cross sections, the pion absorption cross section per nucleon $\sigma_{\rm abs}(W)$ and the usual elastic πN cross sections. The relevant πN reactions read

$$\begin{aligned} &\pi^+ + N \to \pi^+ + N, \quad \pi^+ + n \to \pi^0 + p, \\ &\pi^0 + N \to \pi^0 + N, \quad \pi^0 + p \to \pi^+ + n, \\ &\pi^0 + n \to \pi^- + p, \quad \pi^- + N \to \pi^- + N, \\ &\pi^- + p \to \pi^0 + n, \\ &\pi^{\pm,0} + N \to X, \quad (\pi \notin X)(\to \textit{absorption})\ . \end{aligned}$$

(iv) In the Δ resonance region the πN cross section is dominated by the isospin $I = 3/2$ amplitude such that all elastic cross sections are related to $\sigma_{\pi^+ p}(W)$ by Clebsch-Gordan coefficients. Let

$$\vec{q}_i = \big(n_i(\pi^+), n_i(\pi^0), n_i(\pi^-)\big)^T \quad (4)$$

denote the initial multiplicity distribution of pions in the medium (with fixed pion energy). In

a single elastic scattering this distribution will be modified to a final distribution $\vec{q}_f$:

$$\vec{q}_f = Q\ \vec{q}_i \tag{5}$$

with a known 3×3 matrix Q [1] which follows easily from a Clebsch-Gordan analysis of isospin.

Accordingly, taking into account all possible multiple scatterings the final pion charge distribution is given by

$$\vec{q}_f = M\ \vec{q}_i, \qquad M = \sum_{n=0}^{\infty} P_n Q^n \tag{6}$$

where P_n is the probability that the pion exits the nucleus after exactly n πN scatterings. It should be noted that the matrix Q does not include any absorption which is taken into account by $\sum_{n=0}^{\infty} P_n < 1$. The eigenvalues and eigenvectors of the matrix Q are given by

$$\begin{aligned} \lambda_1 &= 1\ ,\ q_1 = (1,1,1)^T, & (7)\\ \lambda_2 &= \tfrac{5}{6}\ ,\ q_2 = (1,0,-1)^T, & (8)\\ \lambda_3 &= \tfrac{1}{2}\ ,\ q_3 = (1,-2,1)^T & (9) \end{aligned}$$

and one finds immediately the three eigenvalues of the charge exchange matrix M in dependence of the three eigenvalues λ_k of Q

$$f(\lambda_k) = \sum_{n=0}^{\infty} P_n \lambda_k^n\ ,\ k = 1,2,3\ . \tag{10}$$

The connection between the eigenbasis and the canonical basis is given by

$$\begin{aligned} &A_p\,(1-c-d) = \tfrac{1}{3}f(1) + \tfrac{1}{2}f(\tfrac{5}{6}) + \tfrac{1}{6}f(\tfrac{1}{2}),\\ &A_p\,d = \tfrac{1}{3}f(1) - \tfrac{1}{3}f(\tfrac{1}{2}),\\ &A_p\,c = \tfrac{1}{3}f(1) - \tfrac{1}{2}f(\tfrac{5}{6}) + \tfrac{1}{6}f(\tfrac{1}{2}) \end{aligned} \tag{11}$$

or inversely

$$\begin{aligned} A_p &= f(1),\\ c &= \tfrac{1}{3} - \tfrac{1}{2}f(\tfrac{5}{6})/f(1) + \tfrac{1}{6}f(\tfrac{1}{2})/f(1),\\ d &= \tfrac{1}{3}\left[1 - f(\tfrac{1}{2})/f(1)\right]\ . \end{aligned} \tag{12}$$

As already mentioned in the introduction, the Pauli suppression factor of step 1 will be conveniently absorbed into the normalization of the charge exchange matrix M

$$A_p \to A_p = g(W,Q^2)f(1)\ . \tag{13}$$

The function $f(\lambda)$ contains the dynamical details of pion multiple scattering in the nucleus. An outline of how this function is calculated by solving a transport problem for pions in the nucleus will be given in the next section.

2.1. Calculation of $f(\lambda)$

To a very good approximation [1] the three-dimensional problem can be reduced to a one-dimensional transport problem by projecting the forward-hemisphere of the scattered pion onto the forward direction and the backward-hemisphere onto the backward direction [see Eq. (37)]. In this approximation the pion scatters back and forth along its initial direction of motion until it is absorbed or leaves the nucleus. As is explained in more detail in Refs. [1,2] the nucleon density profile along the scattering line can be scaled out of the problem by an appropriate change in length variable such that it is equivalent to consider a *uniform* one-dimensional nuclear medium extending from $x = 0$ to $x = L$ with effective length (optical thickness)

$$L = L(b) = \frac{1}{\rho(0)} \int_{-\infty}^{+\infty} \mathrm{d}z\ \rho(\sqrt{z^2+b^2})\ , \tag{14}$$

where b is the impact parameter. Furthermore, to simplify the discussion we only consider forward scattering of the pions (and also neglect Pauli suppression in this step 2). The solutions taking into account forward and backward scattering, $f_\pm(\lambda)$, can be found in Appendix A of [1]. Apart from being more realistic, they are important for estimates of the background to proton decay by neutrino production of pions in the Δ resonance region [8].

The transport process can be described in terms of the basic probabilities for a pion density to propagate from y to x and to a) interact (scattering or absorption) b) scatter c) be absorbed in $[x, x+dx]$

$$\begin{aligned} &a)\ \langle x|\ P_{\rm tot}\ |y\rangle &=&\ \kappa e^{-\kappa|y-x|}\Theta(y-x) & (15)\\ &b)\ \langle x|\ P_{\rm cex}\ |y\rangle &=&\ \mu\,\langle x|\ P_{\rm tot}\ |y\rangle & (16)\\ &c)\ \langle x|\ P_{\rm abs}\ |y\rangle &=&\ a\,\langle x|\ P_{\rm tot}\ |y\rangle & (17) \end{aligned}$$

with $\kappa = \rho(0)\sigma_{\rm tot}$ ('inverse interaction length'), $\mu = \sigma_{\rm cex}/\sigma_{\rm tot}$ being the probability that the pion

is scattered and $a = \sigma_{\rm abs}/\sigma_{\rm tot}$ the probability that the pion is absorbed (in a single scattering process). The choice of $\rho(0)$ as density of the uniform one-dimensional medium (i.e. $\kappa = \rho(0)\sigma_{\rm tot}$) is related to the normalization of the effective length in Eq. (14). Note also that $\mu + a = 1$ since $\sigma_{\rm tot} = \sigma_{\rm cex} + \sigma_{\rm abs}$. The cross sections for charge exchange (cex) and absorption (abs) will be specified below.

The density of pions in the medium after n scatterings, $|\rho_{\rm in}^{(n)}\rangle$, is related to the initial density $|\rho_{\rm in}^{(0)}\rangle$ by

$$|\rho_{\rm in}^{(n)}\rangle = P_{\rm cex}^n \, |\rho_{\rm in}^{(0)}\rangle \tag{18}$$

where the initial density is normalized to $\langle x|\rho_{\rm in}^{(0)}\rangle = 1/L$ such that the number of pions in the medium is one:

$$N_{\rm in}^{(0)} \equiv \int_0^L {\rm d}x \, \langle x|\rho_{\rm in}^{(0)}\rangle = 1 \; . \tag{19}$$

The density of pions leaving the medium after n scatterings, $|\rho_{\rm out}^{(n)}\rangle$, is given by

$$\begin{aligned} |\rho_{\rm out}^{(n)}\rangle &= (1-P_{\rm tot}) \, |\rho_{\rm in}^{(n)}\rangle & (20) \\ &= (1-P_{\rm tot}) P_{\rm cex}^n \, |\rho_{\rm in}^{(0)}\rangle \; . & (21) \end{aligned}$$

The probability that the pion leaves the medium after exactly n rescatterings is then

$$P_n \equiv N_{\rm out}^{(n)} = \int_0^L \, {\rm d}x \, \langle x|\rho_{\rm out}^{(n)}\rangle \; . \tag{22}$$

The dynamical function $f(\lambda)$ we wish to calculate is then formally given by

$$f(\lambda) = \sum_{n=0}^{\infty} P_n \lambda^n = \int_0^L \, {\rm d}x \, \langle x|\psi_{\rm out}\rangle \tag{23}$$

with

$$|\psi_{\rm out}\rangle := \sum_{n=0}^{\infty} \lambda^n \, |\rho_{\rm out}^{(n)}\rangle \; . \tag{24}$$

Using $1-P_{\rm tot} = 1-\frac{1}{\sigma}+\frac{1}{\sigma}(1-\lambda P_{\rm cex})$ with $\sigma := \lambda\mu$ and $\sum_{n=0}^{\infty} \lambda^n P_{\rm cex}^n = (1-\lambda P_{\rm cex})^{-1} =: 1+F$ we can write $|\psi_{\rm out}\rangle$ as

$$|\psi_{\rm out}\rangle = \left[1 + (1-\tfrac{1}{\sigma})F\right] \, |\rho_{\rm in}^{(0)}\rangle \; . \tag{25}$$

Inserting Eq. (25) into Eq. (23) we find

$$f(\lambda) = 1 + (1-\tfrac{1}{\sigma}) \int_0^L \, {\rm d}x \, {\rm d}y \, \langle x| \, F \, |y\rangle \, \frac{1}{L} \tag{26}$$

where we have inserted the unit operator

$$1 = \int_0^L {\rm d}y \, \; |y\rangle \, \langle y| \tag{27}$$

and used $\langle x|\rho_{\rm in}^{(0)}\rangle = 1/L$.

Using the definition of the operator $1 + F = (1-\lambda P_{\rm cex})^{-1}$ and $(1-\lambda P_{\rm cex})^{-1}(1-\lambda P_{\rm cex}) = 1$ we can furthermore write

$$F = \lambda P_{\rm cex} + F \, \lambda P_{\rm cex} \tag{28}$$

which leads us, using Eqs. (15) and (16), to the transport integral equation

$$f(y) = \sigma(1-e^{-\kappa y}) + \sigma\kappa \int_0^y {\rm d}z \, f(z) e^{-\kappa(y-z)} \tag{29}$$

with

$$f(y) := \int_0^L {\rm d}x \, \langle x| \, F \, |y\rangle \; . \tag{30}$$

The integral equation can be transformed into a differential equation by evaluating $\frac{\rm d}{{\rm d}y}[e^{\kappa y} f(y)]$ for the left and the right side of Eq. (29) resulting in

$$\frac{\rm d}{{\rm d}y} f = \kappa(\sigma-1) f(y) + \kappa\sigma \, , \; f(0) = 0 \; . \tag{31}$$

This differential equation is solved by

$$f(y) = \frac{\sigma}{1-\sigma} \left[1 - h(y)/h(0)\right] \; , \tag{32}$$

where $h(y) = e^{\kappa(\sigma-1)y}$ is a solution of the homogeneous equation $h' = \kappa(\sigma-1)h$. The final solution for the dynamical function $f(\lambda)$ is now easily found to be

$$\begin{aligned} f(\lambda) &= 1 + (1-\tfrac{1}{\sigma}) \frac{1}{L} \int_0^L {\rm d}y \, f(y) \\ &= \frac{1 - e^{-\kappa L(1-\sigma)}}{\kappa L(1-\sigma)} \, , \; \sigma = \lambda\mu \; . & (33) \end{aligned}$$

Finally, we average the solution in Eq. (33) over impact parameters:

$$f(\lambda) = \frac{\int_0^\infty b \; {\rm d}b \; L(b) f(\lambda, L(b))}{\int_0^\infty b \; {\rm d}b \; L(b)} \; . \tag{34}$$

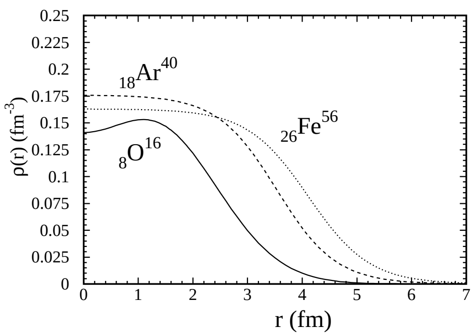

Figure 1. Charge density profiles for oxygen, argon, and iron normalized to $\int \mathrm{d}^3 r \rho(r) = A$.

2.2. ANP model: Input

In this subsection we summarize the input quantities entering the ANP model.

(i) First of all the nucleon density $\rho(r)$ enters the calculation of the effective length $L(b)$ via $L(b) = \frac{1}{\rho(0)} \int \mathrm{d}z \ \rho[r(z,b)]$. The charge density profiles have been determined in electron scattering experiments. For lighter targets like ${}_8O^{16}$ the charge density is parameterized by a 'harmonic oscillator form'

$$\rho(r) = \rho(0) \exp(-r^2/R^2)(1+C\frac{r^2}{R^2}+C_1\frac{r^4}{R^4}) \quad (35)$$

whereas for heavier targets like ${}_{18}Ar^{40}$ and ${}_{26}Fe^{56}$ a 'two parameters Fermi model' is utilized:

$$\rho(r) = \rho(0) \left[1 + \exp\left((r-C)/C_1\right)\right]^{-1} \ . \quad (36)$$

The parameters $\rho(0)$, R, C, and C_1 can be found in Table 1 of [3]. The corresponding charge density profiles $\rho(r)$ are shown in Fig. 1.

(ii) The second input is the usual cross section for elastic πN scattering in the $(3,3)$ resonance region. This region is dominated by the $I = \frac{3}{2}$ amplitude. The elastic cross sections per nucleon for a π^k $(k = \pm, 0)$ in the initial state are given by $Z/A \ \sigma_{\pi^k p} + N/A \ \sigma_{\pi^k n}$. For an isoscalar target $(Z = N)$ and neglecting the $I = \frac{1}{2}$ channel they are independent of the pion charge and will be denoted by $\sigma_{\rm cex}$. Moreover, the elastic cross section $\sigma_{\rm cex}$ is proportional to the cross section $\sigma_{\pi^+ p}(W)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \sigma_{\pi^+ p}(W) \ (1 + 3\cos^2\phi) \ , \quad (37)$$

$$\sigma_{\rm cex} = \tfrac{2}{3}\sigma_{\pi^+ p}(W) \quad (38)$$

with

$$\sigma_{\pi^+ p}(W) = \sigma_{(3,3)}(W) + 20 \text{ mb} \quad (39)$$

where $\sigma_{(3,3)}(W)$ is a resonant cross section which can be found in [1] and the second term is a constant non-resonant background. In Eqs. (37) and (38) we have omitted for simplicity a possible Pauli suppression factor taking into account the Pauli exclusion principle in step 2. Note also, that it is debatable whether such a factor should be included here since we are working in a picture with fixed nucleons.

(iii) The third ingredient is the cross section per nucleon for pion absorption, $\sigma_{\rm abs}(W)$. For this quantity Sternheim and Silbar have published two parametrizations [9,10] which they have extracted from data on single pion production in pA scattering. These parametrizations can also be found in [1]. In Fig. 2 these two absorption cross sections, model (A) and model (B), are shown together with the elastic and the corresponding total cross sections. As can be seen the models (A) and (B) for $\sigma_{\rm abs}$ are quite different in *shape* and *normalization*.

3. Fixing the normalization of $\sigma_{\rm abs}(W)$

The two models for the absorption cross section, model (A) [9] and model (B) [10] result in quite different total amounts of absorbed pions: for example, model (A) predicts that about 19% of the pions will be absorbed in oxygen compared to 43% in model (B).

These numbers have been obtained in an 'averaging approximation' [1] in which the W-dependence of $f(\lambda, W)$ is averaged over the $(3,3)$ region. In this case the dynamical functions (the charge exchange matrix) are mainly sensitive to the region around $W \simeq m_\Delta$ and thus mainly sensitive to the *normalization* of $\sigma_{\rm abs}(W \simeq m_\Delta)$.

In the following we fix this normalization by using experimental data for 1-pion production in

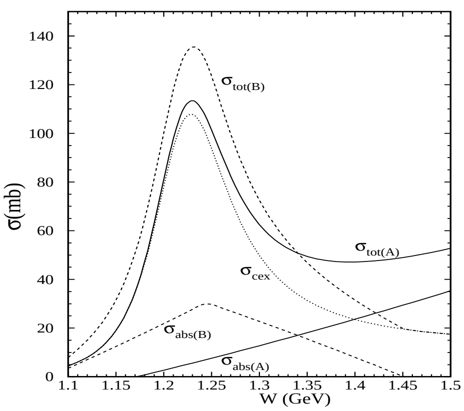

Figure 2. Total, elastic, and absorption cross sections in the $(3,3)$ region.

CC ν_μ-deuteron [$\leftrightarrow$ free case] and ν_μ-neon scattering. These data have been weighted to the *same* atmospheric E_ν spectrum in a paper by Merenyi et al. [11] allowing for direct comparison. In Table 1 of [11] relative charged current populations (w.r.t. the total cross section) are provided for deuteron (D) and neon (Ne). The fractional contributions of the 1-pion production channels (π^+, π^0, π^-) are given by:

$$\mathrm{D}: \vec{r}_i = (0.165, 0.09, 0)^T \qquad (40)$$

$$\mathrm{Ne}: \vec{r}_f = (0.11, 0.05, 0.01)^T \pm (0.014, 0.02, 0.01)^T . \qquad (41)$$

The fractional populations $\vec{r}_i$ and $\vec{r}_f$ are related in the ANP model by:

$$\vec{r}_f = M \vec{r}_i \times K\,, \quad K = \frac{10\sigma_{\mathrm{tot}}(\mathrm{D})}{\sigma_{\mathrm{tot}}(\mathrm{Ne})}\,. \qquad (42)$$

Unfortunately the total cross sections, weighted to an atmospheric neutrino flux, have not been published in [11]. However, the correction factor K should be close to one since the total cross section for Ne is not affected by charge exchange and pion absorption. In the following exercise we assume $K = 1$ and later assign a normalization uncertainty of 3%.

Furthermore, it is not viable to solve $\vec{r}_f \overset{!}{=} M[A_p, d, c]\ \vec{r}_i$ for the three parameters A_p, d, c since the solution *strongly* varies within the errors of $\vec{r}_f$. Instead, we have made the reasonable assumption $0 \le c < d$ which means that the probability for $\pi^- \to \pi^+$ is smaller than the probability for $\pi^0 \to \pi^+$. Under this assumption one can fit the two parameters A_p and d for a fixed value of c as long as $c < d$:

c		A_p	d
$c = 0$	$\to$ Fit:	0.695	0.147
$c = 0.01$	$\to$ Fit:	0.696	0.128
$c = 0.02$	$\to$ Fit:	0.696	0.109
$c = 0.03$	$\to$ Fit:	0.696	0.091
$c = 0.04$	$\to$ Fit:	0.697	0.072
$c = 0.05$	$\to$ Fit:	0.698	0.053

$[\chi^2/d.o.f \simeq 0.4]$

As can be seen the parameter A_p is well constrained: $A_p = 0.696 \pm 0.002$. More conservatively we use in the following $A_p = 0.70 \pm 0.02$ taking into account the normalization uncertainty due to the correction factor K. On the other hand the parameters d and c are correlated and adopt values in the range $c \in [0, 0.05]$, $d \in [0.15, 0.05]$.

Inspecting Eq. (10) we find that $f(\lambda = 1) = \sum_{n=0}^{\infty} P_n = 1 - A$ where A is the fraction of absorbed pions. Therefore, the averaged fraction $\bar{A}$ of absorbed pions is given by $\bar{A} = 1 - \bar{f}(\lambda = 1)$ where $\bar{f}(\lambda)$ is the averaged dynamical function which is related to the above extracted parameter A_p via $A_p = g(\bar{W}, \bar{Q}^2)\bar{f}(1)$. In order to estimate the effect of the Pauli suppression factor we take $\bar{W} \simeq m_\Delta$ and $\bar{Q}^2 \simeq 0.1$ GeV^2 leading to $g(\bar{W}, \bar{Q}^2) = 0.93 \pm 0.05$ [see Table 3 in [3]]. Using these values we finally find $\bar{f}(1) = 0.75 \pm 0.05$ implying that about 25% of the pions have been absorbed in the neon target: $\bar{A} = 0.25 \pm 0.05$.

A fraction of 25% pion absorption can be obtained by renormalizing absorption model (B) [model (A)] by a factor $\simeq 0.3$ [$\simeq 1.4$]. With this renormalization we can compare the ANP model with the above fitted results. The ANP model gives for both renormalized absorption models for a neon target $\bar{f}(1) = 0.75$ (by construction), $d = 0.15$, and $c = 0.05$. This result compares favorably with the fitted values.

4. Dynamics of the ANP model

4.1. Linearisation

The simple solution in Eq. (33) is well-suited for further analytical investigations of pion multiple scattering. For example, the forward solution (33) at $\lambda = 1$,

$$f(\lambda = 1, L, W) = \frac{1 - e^{-\rho_0 L \sigma_{\rm abs}}}{\rho_0 L \sigma_{\rm abs}} , \qquad (43)$$

can be considered in the limit $\rho_0 L \sigma_{\rm abs} \ll 1$ relating the fraction of absorbed pions A to the absorption cross section per nucleon $\sigma_{\rm abs}(W)$:

$$A(L, W) = 1 - f(1, L, W) \simeq \tfrac{1}{2} \rho_0 L \sigma_{\rm abs}(W) . \qquad (44)$$

Averaging $L = L(b)$ over the impact parameters b for oxygen as target material gives $\bar{L} \simeq 1.9R$ with radius $R \simeq 1.833$ fm. The nuclear density for oxygen, $\rho_0 = 0.141$ fm^{-3}, has been taken from Table 1 in [3]. Replacing $\rho_0 L$ by $\rho_0 \bar{L} \simeq 0.05$ mb^{-1} in Eq. (44) and taking into account the averaging over the $(3, 3)$ resonance by the replacement $W \to m_\Delta$ we arrive at the following rule of thumb for the fraction of absorbed pions in oxygen

$$A \simeq 0.025\ \sigma_{\rm abs}(W \simeq m_\Delta)[\rm mb] . \qquad (45)$$

The absorption cross sections can be taken from Fig. 2 and we find the following fractions of absorbed pions:

model	$\sigma_{\rm abs}(W = m_\Delta)$	A
(A)	$\simeq 6.0$ mb	15%
(B)	$\simeq 28.4$ mb	71%
$0.3 \times$ (B)	$\simeq 8.5$ mb	21%

Here the renormalization factor 0.3 for model (B) has been taken from the preceding section. The same result is found by renormalizing model (A) with a factor 1.4. Note also that the 71% for (the original) model (B) is not realistic because the condition $\rho_0 \bar{L} \sigma_{\rm abs} \ll 1$ is not satisfied.

4.2. 'How many' multiple scatterings?

Figure 3 shows the dynamical function $f(\lambda, W)$ for oxygen in dependence of λ for several values of W. Writing $f(\lambda) = \sum_{n=0}^{\infty} P_n \lambda^n$ with the probabilities P_k $(k = 0, 1, 2, \ldots)$ that the pion is observed after k πN scatterings it is obvious that

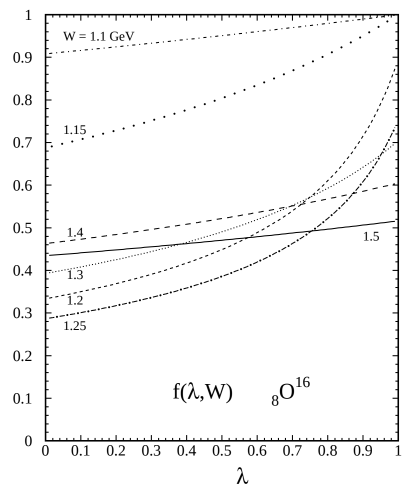

Figure 3. Dynamical function $f(\lambda, W)$ in dependence of λ for several values of W.

$f(\lambda = 0) = P_0$ and $f(\lambda = 1) = 1 - A$ where A is the probability for pion absorption. [Note that $\sum_{k=0}^{\infty} P_k + A = 1$.] It is an interesting question, how large the probabilities $P_n, n > 0$ for multiple scattering are. Qualitatively, a stronger curvature of the function $f(\lambda)$ indicates a higher probability for multiple scattering. As can be seen, in the vicinity of $W = m_\Delta$ the probability that the pion rescatters several time is largest whereas at $W = 1.1$ GeV the curve is almost linear such that only P_0 and P_1 contribute appreciably.

Of course the probabilities P_n can be calculated exactly within the ANP model according to Eq. (22) or by differentiating the solution for $f(\lambda)$ in Eq. (33): $P_n = \frac{1}{n!} \frac{{\rm d}^n}{{\rm d}\lambda^n} f_{|\lambda=0}$.

On the other hand, inspired by the fact that the pion energy remains constant, it is interesting to make the assumption (which is only asymptotically correct) that $P_{k+1} \simeq r P_k$ for $k \geq 1$ or equivalently $P_k \simeq P_1 r^{k-1}$ for $k \geq 1$. Under this assumption $f(\lambda)$ is a geometrical series and is given by the simple solution

$$f(\lambda) = \sum_{n=0}^{\infty} P_n \lambda^n \simeq P_0 + \frac{P_1 \lambda}{1 - r\lambda} . \qquad (46)$$

The constant r can be fixed from $f(\lambda = 1) = 1 - A$

Table 1
Probabilities in % for multiple scattering.

W[GeV]	1.1	1.2	1.25	1.5
A	0.0	11.8	25.7	48.4
P_0	90.8	33.6	28.8	43.4
P_1	8.0	14.1	12.2	6.5
P_2	1.0	10.5	8.9	1.3
P_3	0.13	7.8	6.5	0.26
P_4	...	5.8	4.8	...
P_5		4.3	3.5	
P_6		3.2	2.6	
P_7		2.4	1.9	
P_8		1.8	1.4	
P_9		1.3	1.0	
P_{10}		1.0	0.7	

resulting in $r = 1 - \frac{P_1}{1-A-P_0}$.

Thus, once A, P_0 and P_1 are known, all higher probabilities and the function $f(\lambda)$ can be easily estimated. The result of such a procedure is listed in the following table where P_0, P_1 and A have been determined from Fig. 3.

5. Conclusions

In this article, we have given a basic introduction to the pion multiple scattering model of Adler, Nussinov, and Paschos [1] focusing on the input parameters of the model, particularly on the cross section for pion absorption $\sigma_{\rm abs}(W)$ which is the least well determined ingredient. Using data for ν_μ-deuteron and ν_μ-neon scattering [11] we could fix the normalization of $\sigma_{\rm abs}(W)$ corresponding to a fraction of $(25 \pm 5)\%$ of absorbed pions in neon. The parameters A_p, d, and c have been determined to be $A_p = 0.70 \pm 0.02$, $c \in [0, 0.05]$, and $d \in [0.15, 0.05]$ which compares quite favorably with the predictions of the ANP model for neon.

In order to test and improve the ANP model it will be necessary to make detailed measurements of single pion electroproduction in the region of the $(3,3)$ resonance using different heavy targets and to compare it with the corresponding cross sections on free nucleons [7].

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